From Information Ratio to Hit Rate

A step-by-step derivation, with re-derivation exercises

1. Setup and definitions

Let r denote the active return of a strategy over a single measurement period (a day, week, month, etc.). We make one strong simplifying assumption:

$$r \sim \mathbf{N}(\mu, \sigma^2), \quad \text{i.i.d. across periods.}$$

Define two quantities:

$$\textbf{Hit rate} \quad p = P(r > 0)$$

$$\textbf{Information Ratio} \quad IR_{period} = \mu / \sigma$$

The goal is to express p as a function of IR — and then to express it in terms of the *annualized* IR, which is how the number is conventionally quoted.

2. Hit rate as a normal probability

Standardize r by subtracting its mean and dividing by its standard deviation. The standardized variable $Z = (r - \mu) / \sigma$ is standard normal.

$$P(r > 0) \;=\; P(\,(r - \mu)/\sigma > (0 - \mu)/\sigma\,) \;=\; P(\,Z > -\mu/\sigma\,)$$

By symmetry of the standard normal, $P(Z > -a) = P(Z < a) = \Phi(a)$, where Φ is the standard normal CDF. Substituting:

$$\mathbf{p = \Phi(\mu/\sigma) = \Phi(IR_{period})}$$

That is the whole punchline of step 2: hit rate is the standard normal CDF evaluated at the period IR. If you have a z-table, you have the hit rate.

3. Annualizing the IR

IR is conventionally quoted annually. To use the formula above, we need to convert between annual and per-period IR.

Let there be N measurement periods per year (252 for daily, 52 for weekly, 12 for monthly). The annual return is the sum of N i.i.d. period returns:

$$R_{ann} = r_1 + r_2 + \dots + r_N$$

Means add. Variances add (because the periods are independent). Standard deviations do *not* add — they scale as $\sqrt{N}$. So:

$$E[R_{ann}] = N\mu \qquad SD[R_{ann}] = \sqrt{N} \cdot \sigma$$

Therefore:

$$IR_{ann} \;=\; N\mu \,/\, (\sqrt{N} \cdot \sigma) \;=\; \sqrt{N} \cdot (\mu/\sigma) \;=\; \sqrt{N} \cdot IR_{period}$$

Rearranging gives the conversion you actually use in practice:

$$\mathbf{IR_{period} = IR_{ann} / \sqrt{N}}$$

4. Putting it together

Combine the result of step 2 with the conversion from step 3:

$$\mathbf{p = \Phi(IR_{ann} / \sqrt{N})}$$

For IR_{ann} = 1, the implied hit rate at common frequencies:

Frequency	N	IR_period = 1/√N	Hit rate = Φ(IR_period)
Daily	252	0.0630	52.51%
Weekly	52	0.1387	55.52%
Monthly	12	0.2887	61.36%
Quarterly	4	0.5000	69.15%
Annual	1	1.0000	84.13%

Notice the punchline: an IR of 1 — very good — corresponds to a daily hit rate of barely better than a coin flip. The skill is real but tiny per day; what makes it good is that you get 252 of these small edges per year, and $\sqrt{252} \approx 15.87$.

5. The inverse direction

Sometimes you observe a hit rate (e.g., 53% daily) and want the implied annualized IR. Just invert each step. Apply Φ^{-1} (the inverse normal CDF; `norm.ppf` in scipy, `NORM.S.INV` in Excel) to undo step 2, then multiply by $\sqrt{N}$ to undo step 3:

$$\mathbf{IR_{ann} = \Phi^{-1}(p) \cdot \sqrt{N}}$$

Sanity check with p = 52.51%, daily: $\Phi^{-1}(0.5251) \approx 0.0630$, and $0.0630 \times \sqrt{252} \approx 1.00$.

> Critical: N must match the frequency at which p was measured. Daily hit rate → √252. Monthly → √12. Per-trade → √(trades per year). Mixing these up is the easiest place to make a mistake.

Further reading

- Grinold & Kahn, *Active Portfolio Management* (2nd ed., 1999), chapters 4–7.
- Grinold (1989), *The Fundamental Law of Active Management*, J. Portfolio Management.
- Clarke, de Silva & Thorley (2002), *Portfolio Constraints and the Fundamental Law of Active Management*, FAJ — introduces the Transfer Coefficient.
- Lo (2002), *The Statistics of Sharpe Ratios*, FAJ — the autocorrelation correction.